19 Computational thinking and problem solving

In this chapter, you will learn about

- how to write algorithms to implement linear and binary searches
- the conditions necessary for the use of a binary search
- how the performance of a binary search varies according to the number of data items
- how to write algorithms to implement insertion and bubble sorts
- the use of abstract data types (ADTs) including finding, inserting and deleting items from linked lists, binary trees, stacks and queues
- the key features of a graph and why graphs are used
- how to implement ADTs from other ADTs
- the comparison of algorithms including the use of Big O notation
- how recursion is expressed in a programming language
- when to use recursion
- writing recursive algorithms
- how a compiler implements recursion in a programming language.

19.1 Algorithms

WHAT YOU SHOULD ALREADY KNOW

In Chapter 10 you learnt about Abstract Data Types (ADTs) and searching and sorting arrays. You should also have been writing programs in your chosen programming language (Python, VB or Java). Try the following five questions to refresh your memory before you start to read this chapter.

1 Explain what is meant by the terms

a) stack

b) queue

c) linked list.

2 **a)** Describe how it is possible to implement a stack using an array.

```
// Java program for Hello World
public class Hello {
  public static void main(String[] args) {
    System.out.println("Hello World");
  }
}
```

```
# Python program for Hello World
print ("Hello World")
```

b) Describe how it is possible to implement a queue using an array.

c) Describe how it is possible to implement a linked list using an array.

3 Write pseudocode to search an array of twenty numbers using a linear search.

4 Write pseudocode to sort an array that contains twenty numbers using a bubble sort.

5 Make sure that you can write, compile and run a short program in your chosen programming language. The programs below show the code you need to write the same program in each of three languages.

```
'VB program for Hello World
Module Module1
   Sub Main()
      Console.WriteLine("Hello World")
      Console.ReadKey()
   End Sub
End Module
```

Key terms

Binary search – a method of searching an ordered list by testing the value of the middle item in the list and rejecting the half of the list that does not contain the required value.

Insertion sort – a method of sorting data in an array into alphabetical or numerical order by placing each item in turn in the correct position in the sorted list.

Binary tree – a hierarchical data structure in which each parent node can have a maximum of two child nodes.

Graph – a non-linear data structure consisting of nodes and edges.

Dictionary – an abstract data type that consists of pairs, a key and a value, in which the key is used to find the value.

Big O notation – a mathematical notation used to describe the performance or complexity of an algorithm.

19.1.1 Understanding linear and binary searching methods

Linear search

In Chapter 10, we looked at the linear search method of searching a list. In this method, each element of an array is checked in order, from the lower bound to the upper bound, until the item is found, or the upper bound is reached.

The pseudocode linear search algorithm and identifier table to find if an item is in the populated 1D array myList from Chapter 10 is repeated here.

```
DECLARE myList : ARRAY[0:9] OF INTEGER
DECLARE upperBound : INTEGER
DECLARE lowerBound : INTEGER
DECLARE index : INTEGER
DECLARE item : INTEGER
DECLARE found : BOOLEAN
upperBound ← 9
lowerBound ← 0
OUTPUT "Please enter item to be found"
INPUT item
found ← FALSE
index ← lowerBound
REPEAT
    IF item = myList[index]
      THEN
        found ← TRUE
    ENDIF
    index ← index + 1
UNTIL (found = TRUE) OR (index > upperBound)
IF found
  THEN
    OUTPUT "Item found"
  ELSE
    OUTPUT "Item not found"
ENDIF
```

Identifier	Description
myList	Array to be searched
upperBound	Upper bound of the array
lowerBound	Lower bound of the array
index	Pointer to current array element
item	Item to be found

found	Flag to show when item has been found

Table 19.1

This method works for a list in which the items can be stored in any order, but as the size of the list increases, the average time taken to retrieve an item increases correspondingly.

The Cambridge International AS & A Level Computer Science syllabus requires you to be able to write code in one of the following programming languages: Python, VB and Java. It is very important to practice writing different routines in the programming language of your choice; the more routines you write, the easier it is to write programming code that works.

Here is a simple linear search program written in Python, VB and Java using a FOR loop.

Python

```
#Python program for Linear Search
#create array to store all the numbers
myList = [4, 2, 8, 17, 9, 3, 7, 12, 34, 21]
#enter item to search for
item = int(input("Please enter item to be found "))
found = False
for index in range(len(myList)):
  if(myList[index] == item):
      found = True
if(found):
  print("Item found")
else:
  print("Item not found")
```

VB

```
'VB program for Linear Search
 Module Module1
    Public Sub Main()
        Dim index As Integer
        Dim item As Integer
        Dim found As Boolean
        'Create array to store all the numbers
        Dim myList() As Integer = New Integer() {4, 2, 8, 17, 9, 3, 7, 12, 34, 21}
        'enter item to search for
        Console.Write("Please enter item to be found ")
        item = Integer.Parse(Console.ReadLine())
        For index = 0 To myList.Length - 1
            If (item = myList(index)) Then
                found = True
            End If
        Next
        If (found) Then
            Console.WriteLine("Item found")
        Else : Console.WriteLine("Item not found")
        End If
        Console.ReadKey()
    End Sub
End Module
```

Java

```
//Java program Linear Search
import java.util.Scanner;
public class LinearSearch
{
   public static void main(String args[])
   {
       Scanner myObj = new Scanner(System.in);
   //Create array to store the all the numbers
       int myList[] = new int[] {4, 2, 8, 17, 9, 3, 7, 12, 34, 21};
       int item, index;
       boolean found = false;
       // enter item to search for
   System.out.println("Please enter item to be found ");
   item = myObj.nextInt();
   for (index = 0; index < myList.length - 1; index++)
   {
     if (myList[index] == item)
     {
     found = true;
     }
   }
   if (found)
        {
   System.out.println("Item found");
   }
        else
          {
              System.out.println("Item not found");
            }
        }
}
```

ACTIVITY 19A

Write the linear search in the programming language you are using, then change the code to use a similar type of loop that you used in the pseudocode at the beginning of Section 19.1.1, *Linear search.*

Binary search

A **binary search** is more efficient if a list is already sorted. The value of the middle item in the list is first tested to see if it matches the required item, and the half of the list that does *not* contain the required item is discarded. Then, the next item of the list to be tested is the middle item of the half of the list that was kept. This is repeated until the required item is found or there is nothing left to test.

For example, consider a list of the letters of the alphabet.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

To find the letter W using a linear search there would be 23 comparisons.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-			
W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W			
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23			

Figure 19.1 Linear search showing all the comparisons

To find the letter W using a binary search there could be just three comparisons.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
												-							-			-			
												W							W			W			
												1							2			3			

Figure 19.2 Binary search showing all the comparisons

ACTIVITY 19B

Check how many comparisons for each type of search it takes to find the letter D. Find any letters where the linear search would take less comparisons than the binary search.

A binary search usually takes far fewer comparisons than a linear search to find an item in a list. For example, if a list had 1024 elements, the maximum number of comparisons for a binary search would be 16, whereas a linear search could take up to 1024 comparisons.

Here is the pseudocode for the binary search algorithm to find if an item is in the populated 1D array myList. The identifier table is the same as the one used for the linear search.

```
DECLARE myList : ARRAY[0:9] OF INTEGER
DECLARE upperBound : INTEGER
DECLARE lowerBound : INTEGER
DECLARE index : INTEGER
DECLARE item : INTEGER
DECLARE found : BOOLEAN
upperBound ← 9
lowerBound ← 0
OUTPUT "Please enter item to be found"
INPUT item
found ← FALSE
REPEAT
    index ← INT ( (upperBound + lowerBound) / 2 )
    IF item = myList[index]
      THEN
        found ← TRUE
    ENDIF
    IF item > myList[index]
      THEN
        lowerBound ← index + 1
    ENDIF
    IF item < myList[index]
      THEN
        upperBound ← index - 1
    ENDIF
UNTIL (found = TRUE) OR (lowerBound = upperBound)
IF found
  THEN
    OUTPUT "Item found"
  ELSE
    OUTPUT "Item not found"
ENDIF
```

Identifier	Description
myList	Array to be searched
upperBound	Upper bound of the array
lowerBound	Lower bound of the array

index	Pointer to current array element
item	Item to be found
found	Flag to show when item has been found

Table 19.2

The code structure for a binary search is very similar to the linear search program shown for each of the programming languages. You will need to populate myList before searching for an item, as well as the variables found, lowerBound and upperBound.

You will need to use a conditional loop like those shown in the table below.

Loop	Language
`while (not found) and (lowerBound != lowerBound):`	Python uses a condition to repeat the loop at the start of the loop

`Do`
`:`
`:`
`Loop Until (found) Or (lowerBound = upperBound)`
	VB uses a condition to stop the loop at the end of the loop

`Do {`
`:`
`:`
`}`
`while ((!found) && (upperBound != lowerBound));`
	Java uses a condition to repeat the loop at the end of the loop

Table 19.3

You will need to use If statements like those shown in the table below to test if the item is found, or to decide which part of myList to use next, and to update the upperBound or lowerBound accordingly.

If	Language

`index = (upperBound + lowerBound)//2`
`if(myList[index] == item):`
`found = True`
`if item > myList[index]:`
`lowerBound = index + 1`
`if item < myList[index]:`
`upperBound = index - 1`
	Python using integer division

	VB using integer division

`index = (upperBound + lowerBound)\2`
`  If (item = myList(index)) Then`
`  found = True`
`  End If`
`  If item > myList(index) Then`
`  lowerBound = index + 1`
`  End if`
`  If item < myList(index) Then`
`  upperBound = index -1`
`  End if`	

`index = (upperBound + lowerBound) / 2;`
`  if (myList[index] == item)`
`  {`
`  found = true;`
`  }`
`  if (item > myList[index])`
`  {`
`  lowerBound = index + 1;`
`  }`
`  if (item < myList[index])`
`  {`
`  upperBound = index - 1;`
`  }`
	Java automatic integer division

Table 19.4

ACTIVITY 19C

In your chosen programming language, write a short program to complete the binary search.

Use this sample data:

16, 19, 21, 27, 36, 42, 55, 67, 76, 89

Search for the values 19 and 77 to test your program.

19.1.2 Understanding insertion and bubble sorting methods

Bubble sort

In Chapter 10, we looked at the bubble sort method of sorting a list. This is a method of sorting data in an array into alphabetical or numerical order by comparing adjacent items and swapping them if they are in the wrong order.

The bubble sort algorithm and identifier table to sort the populated 1D array myList from Chapter 10 is repeated here.

```
DECLARE myList : ARRAY[0:8] OF INTEGER
DECLARE upperBound : INTEGER
DECLARE lowerBound : INTEGER
DECLARE index : INTEGER
DECLARE swap : BOOLEAN
DECLARE temp : INTEGER
DECLARE top : INTEGER
upperBound ← 8
lowerBound ← 0
top ← upperBound
REPEAT
   FOR index = lowerBound TO top - 1
         Swap ← FALSE
         IF myList[index] > myList[index + 1]
           THEN
           temp ← myList[index]
           myList[index] ← myList[index + 1]
           myList[index + 1] ← temp
           swap ← TRUE
         ENDIF
    NEXT
    top ← top -1
UNTIL (NOT swap) OR (top = 0)
```

Identifier	Description
myList	Array to be searched
upperBound	Upper bound of the array
lowerBound	Lower bound of the array
index	Pointer to current array element
swap	Flag to show when swaps have been made
top	Index of last element to compare
temp	Temporary storage location during swap

Table 19.5

Here is a simple bubble sort program written in Python, VB and Java, using a pre-condition loop and a FOR loop in Python and post-condition loops and FOR loops in VB and Java.

Python

Pre-condition loop

```
#Python program for Bubble Sort
myList = [70,46,43,27,57,41,45,21,14]
top = len(myList)
swap = True
while (swap) or (top > 0):
        swap = False
        for index in range(top - 1):
                if myList[index] > myList[index + 1]:
                        temp = myList[index]
                        myList[index] = myList[index + 1]
                        myList[index + 1] = temp
                        swap = True
        top = top - 1
#output the sorted array
print(myList)
```

VB

```
'VB program for bubble sort
 Module Module1
    Sub Main()
        Dim myList() As Integer = New Integer() {70, 46, 43, 27, 57, 41, 45, 21, 14}
        Dim index, top, temp As Integer
        Dim swap As Boolean
        top = myList.Length - 1
        Do
            swap = False
            For index = 0 To top - 1 Step 1
                If myList(index) > myList(index + 1) Then
                    temp = myList(index)
                    myList(index) = myList(index + 1)
                    myList(index + 1) = temp
                    swap = True
                End If
            Next
            top = top - 1
        Loop Until (Not swap) Or (top = 0)
        'output the sorted array
        For index = 0 To myList.Length - 1
            Console.Write(myList(index) & " ")
        Next
        Console.ReadKey() 'wait for keypress
    End Sub
End Module
```

Post-condition loop

Java

```
// Java program for Bubble Sort
class BubbleSort
{
 public static void main(String args[])
  {
      int myList[] = {70, 46, 43, 27, 57, 41, 45, 21, 14};
      int index, top, temp;
      boolean swap;
        top = myList.length;
        do {
              swap = false;
              for (index = 0; index < top - 1; index++)
              {
                if (myList[index] > myList[index + 1])
                {
                   temp = myList[index];
                   myList[index] = myList[index + 1];
                   myList[index + 1] = temp;
                     swap = true;
                     }
              }
              top = top - 1;
      }
      while ((swap) || (top > 0));
      // output the sorted array
        for (index = 0; index < myList.length; index++)
             System.out.print(myList[index] + " ");
        System.out.println();
  }
}
```

Post-condition loop

Insertion sort

The bubble sort works well for short lists and partially sorted lists. An insertion sort will also work well for these types of list. An **insertion sort** sorts data in a list into alphabetical or numerical order by placing each item in turn in the correct position in a sorted list. An insertion sort works well for incremental sorting, where elements are added to a list one at a time over an extended period while keeping the list sorted.

Here is the pseudocode and the identifier table for the insertion sort algorithm sorting the populated 1D array myList.

```
DECLARE myList : ARRAY[0:8] OF INTEGER
DECLARE upperBound : INTEGER
DECLARE lowerBound : INTEGER
DECLARE index : INTEGER
DECLARE key : BOOLEAN
DECLARE place : INTEGER
upperBound ← 8
lowerBound ← 0
FOR index ← lowerBound + 1 TO upperBound
    key ← myList[index]
    place ← index - 1
    IF myList[place] > key
      THEN
        WHILE place >= lowerBound AND myList[place] > key
            temp ← myList[place + 1]
            myList[place + 1] ← myList[place]
            myList[place] ← temp
            place  ← place - 1
        ENDWHILE
        myList[place + 1] ← key
    ENDIF
NEXT index
```

Identifier	Description
myList	Array to be searched
upperBound	Upper bound of the array
lowerBound	Lower bound of the array
index	Pointer to current array element
key	Element being placed
place	Position in array of element being moved

Table 19.6

Figure 19.3 shows the changes to the 1D array myList as the insertion sort is completed.

myList	Index of element being checked												
	1		2	3	4		5	6		7		8	
[0]	27	19	19	19	19	16	16	16	16	16	16	16	16
[1]	19	27	27	27	27	19	19	19	19	19	16	16	16
[2]	36	36	36	36	36	27	27	27	21	21	19	19	19
[3]	42	42	42	42	42	36	36	36	27	27	21	21	21
[4]	16	16	16	16	16	42	42	42	36	36	27	27	27
[5]	89	89	89	89	89	89	89	89	42	42	36	36	36
[6]	21	21	21	21	21	21	21	21	89	89	42	42	42
[7]	16	16	16	16	16	16	16	16	16	16	89	89	55
[8]	55	55	55	55	55	55	55	55	55	55	55	55	89

Figure 19.3

The element shaded blue is being checked and placed in the correct position. The elements shaded yellow are the other elements that also need to be moved if the element being checked is out of position. When sorting the same array, myList, the insert sort made 21 swaps and the bubble sort shown in Chapter 10 made 38 swaps. The insertion sort performs better on partially sorted lists because, when each element is found to be in the wrong order in the list, it is moved to approximately the right place in the list. The bubble sort will only swap the element in the wrong order with its neighbour.

As the number of elements in a list increases, the time taken to sort the list increases. It has been shown that, as the number of elements increases, the performance of the bubble sort deteriorates faster than the insertion sort.

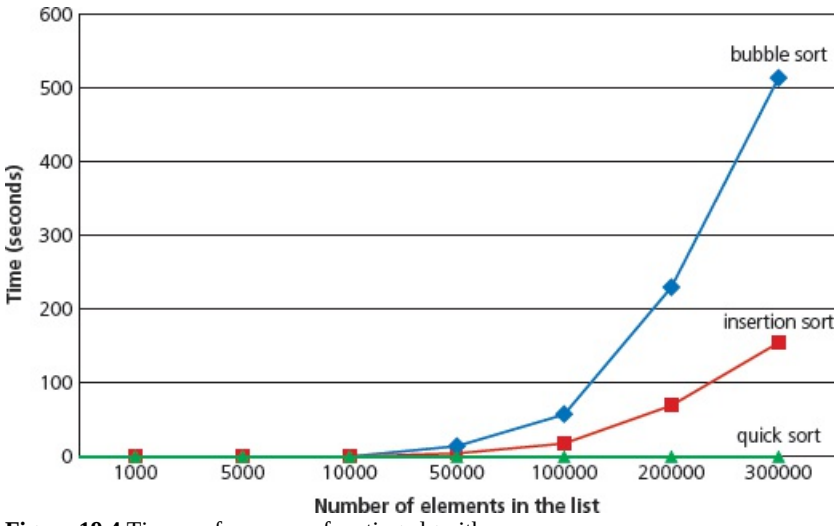


Figure 19.4 Time performance of sorting algorithms

The code structure for an insertion sort in each of the programming languages is very similar to the bubble sort program. You will need to assign values to lowerBound and upperBound and use a nested loop like those shown in the table below.

Nested loop	Language

for index in range(lowerBound + 1, upperBound):
    key = myList[index]
    place = index -1
    if myList[place] > key:
        while place >= lowerBound and myList[place] > key:
            temp = myList[place + 1]
            myList[place + 1] = myList [place]
            myList[place] = temp
            place = place -1
        myList[place + 1] = key
	Python

For index = lowerBound + 1 To upperBound
    myKey = myList(index)
    place = index - 1
    If myList(place) > myKey Then
        While (place >= lowerBound) And (myList(place) > myKey)
            temp = myList(place + 1)
            myList(place + 1) = myList(place)
            myList(place) = temp
            place = place - 1
        End While
        myList(place + 1) = myKey
    End If
Next
	VB cannot use key as a variable

for (index = lowerBound + 1; index < upperBound; index++)
    {
        key = myList[index];
        place = index - 1;
        if (myList[place] > key)
        {
        do
        {
        temp = myList[place + 1];
        myList[place + 1] = myList[place];
        myList[place] = temp;
        place = place - 1
        }
        while ((place >= lowerBound) || (myList[place + 1] > key));
        myList[place + 1] = key;
        }
    }
	Java

Table 19.7

ACTIVITY 19D

In your chosen programming language write a short program to complete the insertion sort.

EXTENSION ACTIVITY 19A

There are many other more efficient sorting algorithms. In small groups, investigate different sorting algorithms, finding out how the method works and the efficiency of that method. Share your results.

19.1.3 Understanding and using abstract data types (ADTs)

Abstract data types (ADTs) were introduced in Chapter 10. Remember that an ADT is a collection of data and a set of operations on that data. There are several operations that are essential when using an ADT

- finding an item already stored
- adding a new item
- deleting an item.

We started considering the ADTs stacks, queues and linked lists in Chapter 10. If you have not already done so, read Section 10.4 to ensure that you are ready to work with these data structures. Ensure that you can write algorithms to set up then add and remove items from stacks and queues.

Stacks

In Chapter 10, we looked at the data and the operations for a stack using pseudocode. You will need to be able to write a program to implement a stack. The data structures and operations required to implement a similar stack using a fixed length integer array and separate sub routines for the push and pop operations are set out below in each of the three prescribed programming languages. If you are unsure how the operations work, look back at Chapter 10.

Stack data structure	Languag

`stack = [None for index in range(0,10)]`
`basePointer = 0`
`topPointer = -1`
`stackFull = 10`
`item = None`
	Python
	empty stack wit no elements

`Public Dim stack() As Integer = {Nothing, Nothing, Nothing, Nothing, Nothing, Nothing, Nothing, Nothing, Nothing, Nothing}`
`Public Dim basePointer As Integer = 0`
`Public Dim topPointer As Integer = -1`
`Public Const stackFull As Integer = 10`
`Public Dim item As Integer`
	VB
	empty stack wit no elements and variables set to public fo subroutin access

	Java

`public static int stack[] = new int[] {0, 0, 0, 0, 0, 0, 0, 0, 0, 0};`
`public static int basePointer = 0;`
`public static int topPointer = -1;`
`public static final int stackFull = 10;`
`public static int item;`
	empty stack wit no elements and variables set to public fo subroutin access

Table 19.8

Stack pop operation	Language

`def pop():`
`  global topPointer, basePointer, item`
`  if topPointer == basePointer -1:`
`  print("Stack is empty,cannot pop")`
`  else:`
`  item = stack[topPointer]`
`  topPointer = topPointer -1`
	Python
	global used within subroutine to access variables
	topPointer points to the top of the stack

`Sub pop()`
`  If topPointer = basePointer - 1 Then`
`  Console.WriteLine("Stack is empty, cannot pop")`
`  Else`
`  item = stack(topPointer)`
`  topPointer = topPointer - 1`
`  End If`
`End Sub`
	VB
	topPointer points to the top of the stack

	Java topPointer points to the top of the stack

```
static void pop()
{
      if (topPointer == basePointer - 1)
                  System.out.println("Stack is empty,cannot pop");
      else
      {
            item = stack[topPointer - 1];
              topPointer = topPointer - 1;
      }
}
```

Table 19.9

Stack push operation	Language

`def push(item):`
`  global topPointer`
`  if topPointer < stackFull - 1:`
`  topPointer = topPointer + 1`
`  stack[topPointer] = item`
`  else:`
`  print("Stack is full, cannot push")`
	Python

`Sub push(ByVal item)`
`  If topPointer < stackFull - 1 Then`
`  topPointer = topPointer + 1`
`  stack(topPointer) = item`
`  Else`
`  Console.WriteLine("Stack is full, cannot push")`
`  End if`
`End Sub`
	VB

`static void push(int item)`
`{`
`  if (topPointer < stackFull - 1)`
`  {`
`  topPointer = topPointer + 1;`
`  stack[topPointer] = item;`
`  }`
`  else`
`  System.out.println("Stack is full, cannot push");`
`}`
	Java

Table 19.10

ACTIVITY 19E

In your chosen programming language, write a program using subroutines to implement a stack with 10 elements. Test your program by pushing two integers 7 and 32 onto the stack, popping these integers off the stack, then trying to remove a third integer, and by pushing the integers 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 onto the stack, then trying to push 11 on to the stack.

Queues

In Chapter 10, we looked at the data and the operations for a circular queue using pseudocode. You will need to be able to write a program to implement a queue. The data structures and operations required to implement a similar queue using a fixed length integer array and separate sub routines for the enqueue and dequeue operations are set out below in each of the three programing languages. If you are unsure how the operations work, look back at Chapter 10.

Queue data structure	Languag

`queue = [None for index in range(0,10)]`
`frontPointer = 0`
`rearPointer = -1`
`queueFull = 10`
`queueLength = 0`
	Python
	empty queue with no items

`Public Dim queue() As Integer = {Nothing, Nothing, Nothing, Nothing, Nothing, Nothing, Nothing, Nothing, Nothing, Nothing}`
`Public Dim frontPointer As Integer = 0`
`Public Dim rearPointer As Integer = -1`
`Public Const queueFull As Integer = 10`
`Public Dim queueLength As Integer = 0`
`Public Dim item As Integer`
	VB
	empty queue with no items and variables, set to public for subroutin access

`public static int queue[] = new int[] {0, 0, 0, 0, 0, 0, 0, 0, 0, 0};`
`public static int frontPointer = 0;`
`public static int rearPointer = -1;`
`public static final int queueFull = 10;`
`public static int queueLength = 0;`
`public static int item;`
	Java
	empty queue with no elements and variables, set to public for subroutin access

Table 19.11

Queue enqueue (add item to queue) operation	Language

def enQueue(item):
  global queueLength, rearPointer
  if queueLength < queueFull:
  if rearPointer < len(queue) - 1:
  rearPointer = rearPointer + 1
  else:
  rearPointer = 0
  queueLength = queueLength + 1
  queue[rearPointer] = item
  else:
  print("Queue is full, cannot enqueue")
	Python
	global used within subroutine to access variables
	If the rearPointer is pointing to the last element of the array and the queue is not full, the item is stored in the first element of the array

Sub enQueue(ByVal item)
  If queueLength < queueFull Then
  If rearPointer < queue.length - 1 Then
  rearPointer = rearPointer + 1
  Else
  rearPointer = 0
  End If
  queueLength = queueLength + 1
  queue(rearPointer) = item
  Else
  Console.WriteLine("Queue is full, cannot enqueue")
  End If
End Sub
	VB
	If the rearPointer is pointing to the last element of the array and the queue is not full, the item is stored in the first element of the array

	Java If the rearPointer is pointing to the last element of the array and the queue is not full, the item is stored in the first

static void enQueue(int item)
{
  if (queueLength < queueFull)
  {
  if (rearPointer < queue.length - 1)
  rearPointer = rearPointer + 1;
  else
  rearPointer = 0;
  queueLength = queueLength + 1;
  queue[rearPointer] = item;
  }
  else
  System.out.println("Queue is full, cannot enqueue");
};
	element of the array

Table 19.12

Queue dequeue (remove item from queue) operation	Language

def deQueue():
  global queueLength, frontPointer, item
  if queueLength == 0:
  print("Queue is empty,cannot dequeue")
  else:
  item = queue[frontPointer]
  if frontPointer == len(queue) - 1:
  frontPointer = 0
  else:
  frontPointer = frontPointer + 1
  queueLength = queueLength -1
	Python
	If the frontPointer points to the last element in the array and the queue is not empty, the pointer is updated to point at the first item in the array rather than the next item in the array

	VB If the frontPointer points to the last element in the array and the queue is not

Sub deQueue()
  If queueLength = 0 Then
  Console.WriteLine("Queue is empty, cannot dequeue")
  Else
  item = queue(frontPointer)
  If frontPointer = queue.length - 1 Then
  frontPointer = 0
  Else
  frontPointer = frontPointer + 1
  End if
  queueLength = queueLength - 1
  End If
End Sub
	empty, the pointer is updated to point at the first item in the array rather than the next item in the array

static void deQueue()
{
  if (queueLength == 0)
  System.out.println("Queue is empty,cannot dequeue");
  else
  {
  item = queue[frontPointer];
  if (frontPointer == queue.length - 1)
  frontPointer = 0;
  else
  frontPointer = frontPointer + 1;
  queueLength = queueLength - 1;
  }
}
	Java
	If the frontPointer points to the last element in the array and the queue is not empty, the pointer is updated to point at the first item in the array rather than the next item in the array

Table 19.13

ACTIVITY 19F

In your chosen programming language, write a program using subroutines to implement a queue with 10 elements. Test your program by adding two integers 7 and 32 to the queue, removing these integers from the queue, then trying to remove a third integer, and by adding the integers 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 to the queue then trying to add 11 to the queue.

Linked lists

Finding an item in a linked list

In Chapter 10, we looked at defining a linked list as an ADT; now we need to consider writing

algorithms using a linked list. Here is the declaration algorithm and the identifier table from Chapter 10.

```
DECLARE mylinkedList ARRAY[0:11] OF INTEGER
DECLARE myLinkedListPointers ARRAY[0:11] OF INTEGER
DECLARE startPointer : INTEGER
DECLARE heapStartPointer : INTEGER
DECLARE index : INTEGER
heapStartPointer ← 0
startPointer ← -1 // list empty
FOR index ← 0 TO 11
    myLinkedListPointers[index] ← index + 1
NEXT index
// the linked list heap is a linked list of all the
spaces in the linked list, this is set up when the
linked list is initialised
myLinkedListPointers[11] ← -1
// the final heap pointer is set to -1 to show no
further links
```

The above code sets up a linked list ready for use. The identifier table is below.

Identifier	Description
myLinkedList	Linked list to be searched
myLinkedListPointers	Pointers for linked list
startPointer	Start of the linked list
heapStartPointer	Start of the heap
index	Pointer to current element in the linked list

Table 19.14

Figure 19.5 below shows an empty linked list and its corresponding pointers.

		myLinkedList	myLinkedListPointers
heapStartPointer →	[0]		1
	[1]		2
	[2]		3
	[3]		4
startPointer = -1	[4]		5
	[5]		6
	[6]		7
	[7]		8
	[8]		9
	[9]		10
	[10]		11
	[11]		–1

Figure 19.5

Figure 19.6 below shows a populated linked list and its corresponding pointers.

		myLinkedList	myLinkedListPointers
	[0]	27	–1
	[1]	19	0
	[2]	36	1
	[3]	42	2
startPointer →	[4]	16	3
heapStartPointer →	[5]		6
	[6]		7
	[7]		8
	[8]		9
	[9]		10
	[10]		11
	[11]		–1

Figure 19.6

The algorithm to find if an item is in the linked list myLinkedList and return the pointer to the item if found or a null pointer if not found, could be written as a function in pseudocode as shown below.

```
DECLARE itemSearch : INTEGER
DECLARE itemPointer : INTEGER
CONSTANT nullPointer = -1
FUNCTION find(itemSearch) RETURNS INTEGER
DECLARE found : BOOLEAN
itemPointer ← startPointer
found ← FALSE
    WHILE (itemPointer <> nullPointer) AND NOT found DO
        IF myLinkedList[itemPointer] = itemSearch
          THEN
            found ← TRUE
          ELSE
            itemPointer ← myLinkedListPointers[itemPointer]
        ENDIF
    ENDWHILE
RETURN itemPointer
// this function returns the item pointer of the value found or -1 if the
item is not found
```

The following programs use a function to search for an item in a populated linked list.

Python

```
#Python program for finding an item in a linked list
myLinkedList = [27, 19, 36, 42, 16, None, None, None, None, None, None, None]
myLinkedListPointers = [-1, 0, 1, 2, 3 ,6 ,7 ,8 ,9 ,10 ,11, -1]
startPointer = 4
nullPointer = -1

def find(itemSearch):
    found = False
    itemPointer = startPointer
    while itemPointer != nullPointer and not found:
        if myLinkedList[itemPointer] == itemSearch:
            found = True
        else:
            itemPointer = myLinkedListPointers[itemPointer]
    return itemPointer

#enter item to search for
item = int(input("Please enter item to be found "))
result = find(item)
if result != -1:
    print("Item found")
else:
    print("Item not found")
```

Populating the linked list

Defining the find function

Calling the find function

VB

```
'VB program for finding an item in a linked list
Module Module1
    Public Dim startPointer As Integer = 4
    Public Const nullPointer As Integer = -1
      Public Dim item As Integer
      Public Dim itemPointer As Integer
      Public Dim result As Integer
      Public Dim myLinkedList() As Integer = {27, 19, 36, 42, 16,
        Nothing, Nothing, Nothing, Nothing, Nothing, Nothing, Nothing}
      Public Dim myLinkedListPointers() As Integer = {-1, 0, 1, 2,
        3, 6, 7, 8, 9, 10, 11, -1}
         Public Sub Main()
         'enter item to search for
            Console.Write("Please enter item to be found ")
            item = Integer.Parse(Console.ReadLine())
            result = find(item)
            If result <> -1 Then
               Console.WriteLine("Item found")
            Else
               Console.WriteLine("Item not found")
            End If
            Console.ReadKey()
         End Sub

    Function find(ByVal itemSearch As Integer) As Integer
         Dim found As Boolean = False
         itemPointer = startPointer
         While (itemPointer <> nullPointer) And Not found
            If itemSearch = myLinkedList(itemPointer) Then
            found = True
          Else
            itemPointer = myLinkedListPointers(itemPointer)
          End If
         End While
         Return itemPointer
    End Function
End Module
```

Populating the linked list

Calling the find function

Defining the find function

Java

```
//Java program for finding an item in a linked list
import java.util.Scanner;
class LinkedListAll
{
  public static int myLinkedList[] = new int[] {27, 19, 36, 42, 16, 0,
    0, 0, 0, 0, 0, 0};
  public static int myLinkedListPointers[] = new int[] {-1, 0, 1, 2,
    3, 6, 7, 8, 9, 10, 11, -1};
  public static int startPointer = 4;
  public static final int nullPointer = -1;
  static int find(int itemSearch)
  {
  boolean found = false;
     int itemPointer = startPointer;
     do
     {
          if (itemSearch == myLinkedList[itemPointer])
          {
             found = true;
          }
          else
          {
              itemPointer = myLinkedListPointers[itemPointer];
          }
     }
     while ((itemPointer != nullPointer) && !found);
     return itemPointer;

  }
   public static void main(String args[])
  {
     Scanner input = new Scanner(System.in);
   System.out.println("Please enter item to be found ");
   int item = input.nextInt();
   int result = find(item);
   if (result != -1 )
     {
   System.out.println("Item found");
  }
    else
     {
          System.out.println("Item not found");
        }
     }
  }
```

Populating the linked list

Defining the find function

Calling the find function

The trace table below shows the algorithm being used to search for 42 in myLinkedList.

startPointer	itemPointer	searchItem
Already set to 4	4	42
	3	

Table 19.15 Trace table

ACTIVITY 19G

In the programming language of your choice, use the code given to write a program to set up the populated linked list and find an item stored in it.

Inserting items into a linked list

The algorithm to insert an item in the linked list myLinkedList could be written as a procedure in pseudocode as shown below.

```
DECLARE itemAdd : INTEGER
DECLARE startPointer : INTEGER
DECLARE heapstartPointer : INTEGER
DECLARE tempPointer : INTEGER
CONSTANT nullPointer = -1
PROCEDURE linkedListAdd(itemAdd)
  // check for list full
  IF heapStartPointer = nullPointer
    THEN
      OUTPUT "Linked list full"
    ELSE
      // get next place in list from the heap
      tempPointer ← startPointer // keep old start pointer
      startPointer ← heapStartPointer  // set start pointer to next position in heap
      heapStartPointer ← myLinkedListPointers[heapStartPointer] // reset heap start pointer
      myLinkedList[startPointer] ← itemAdd // put item in list
      myLinkedListPointers[startPointer] ← tempPointer  // update linked list pointer
  ENDIF
ENDPROCEDURE
```

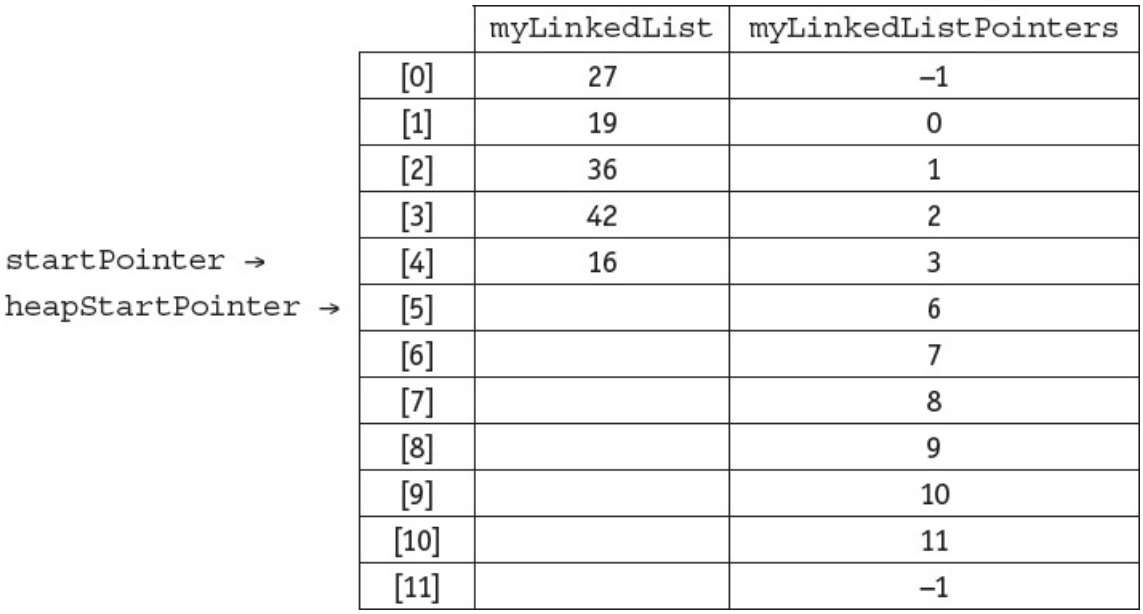
Here is the identifier table.

Identifier	Description
startPointer	Start of the linked list

heapStartPointer	Start of the heap
nullPointer	Null pointer set to -1
itemAdd	Item to add to the list
tempPointer	Temporary pointer

Table 19.16

Figure 19.7 below shows the populated linked list and its corresponding pointers again.



	myLinkedList	myLinkedListPointers
[0]	27	–1
[1]	19	0
[2]	36	1
[3]	42	2
[4]	16	3
[5]		6
[6]		7
[7]		8
[8]		9
[9]		10
[10]		11
[11]		–1

Figure 19.7

The trace table below shows the algorithm being used to add 18 to myLinkedList.

startPointer	heapStartPointer	itemAdd	tempPointer
Already set to 4	Already set to 5	18	
5	6		4

Table 19.17 Trace table

The linked list, myLinkedList, will now be as shown below.

		myLinkedList	myLinkedListPointers
	[0]	27	–1
	[1]	19	0
	[2]	36	1
	[3]	42	2
	[4]	16	3
startPointer →	[5]	18	4
heapStartPointer →	[6]		7
	[7]		8
	[8]		9
	[9]		10
	[10]		11
	[11]		–1

Figure 19.8

The following procedure adds an item to a linked list.

Python

```
def insert(itemAdd):
    global startPointer
    if heapStartPointer == nullPointer:
        print("Linked List full")
    else:
        tempPointer = startPointer
        startPointer = heapStartPointer
heapStartPointer = myLinkedListPointers[heapStartPointer]
        myLinkedList[startPointer] = itemAdd
        myLinkedListPointers[startPointer] = tempPointer
```

Adjusting the pointers and adding the item

VB

```
Sub insert (ByVal itemAdd)
    Dim tempPointer As Integer
    If heapStartPointer = nullPointer Then
       Console.WriteLine("Linked List full")
    Else
       tempPointer = startPointer
       startPointer = heapStartPointer
       heapStartPointer = myLinkedListPointers(heapStartPointer)
       myLinkedList(startPointer) = itemAdd
       myLinkedListPointers(startPointer) = tempPointer
    End if
End Sub
```

Adjusting the pointers and adding the item

Java

```
static void insert(int itemAdd)
{
    if (heapStartPointer == nullPointer)
       System.out.println("Linked List is full");
    else
    {
int tempPointer = startPointer;
       startPointer = heapStartPointer;
       heapStartPointer = myLinkedListPointers[heapStartPointer];
       myLinkedList[startPointer] = itemAdd;
       myLinkedListPointers[startPointer] = tempPointer;      }
}
```

ACTIVITY 19H

Use the algorithm to add 25 to myLinkedList. Show this in a trace table and show myLinkedList once 25 has been added. Add the insert procedure to your program, add code to input an item, add this item to the linked list then print out the list and the pointers before and after the item was added.

Deleting items from a linked list

The algorithm to delete an item from the linked list myLinkedList could be written as a procedure in pseudocode as shown below.

```
DECLARE itemDelete : INTEGER
DECLARE oldIndex : INTEGER
DECLARE index : INTEGER
DECLARE startPointer : INTEGER
DECLARE heapStartPointer : INTEGER
DECLARE tempPointer : INTEGER
CONSTANT nullPointer = -1
PROCEDURE linkedListDelete(itemDelete)
  // check for list empty
  IF startPointer = nullPointer
    THEN
      OUTPUT "Linked list empty"
    ELSE
      // find item to delete in linked list
      index ← startPointer
      WHILE myLinkedList[index] <> itemDelete AND
        (index <> nullPointer) DO
          oldIndex ← index
          index ← myLinkedListPointers[index]
      ENDWHILE
      IF index = nullPointer
        THEN
          OUTPUT "Item ", itemDelete, " not found"
        ELSE
        // delete the pointer and the item
         tempPointer ← myLinkedListPointers[index]
         myLinkedListPointers[index] ← heapStartPointer
         heapStartPointer ← index
         myLinkedListPointers[oldIndex] ← tempPointer
     ENDIF
  ENDIF
ENDPROCEDURE
```

Here is the identifier table.

Identifier	Description
startPointer	Start of the linked list

heapStartPointer	Start of the heap
nullPointer	Null pointer set to −1
index	Pointer to current list element
oldIndex	Pointer to previous list element
itemDelete	Item to delete from the list
tempPointer	Temporary pointer

Figure 19.18

The trace table below shows the algorithm being used to delete 36 from myLinkedList.

startPointer	heapStartPointer	itemDelete	index	oldIndex	tempPointer
Already set to 4	Already set to 5	36	4	4	
			3	3	
			2		
	2				1

Table 19.19 Trace table

The linked list, myLinkedList, will now be as follows.

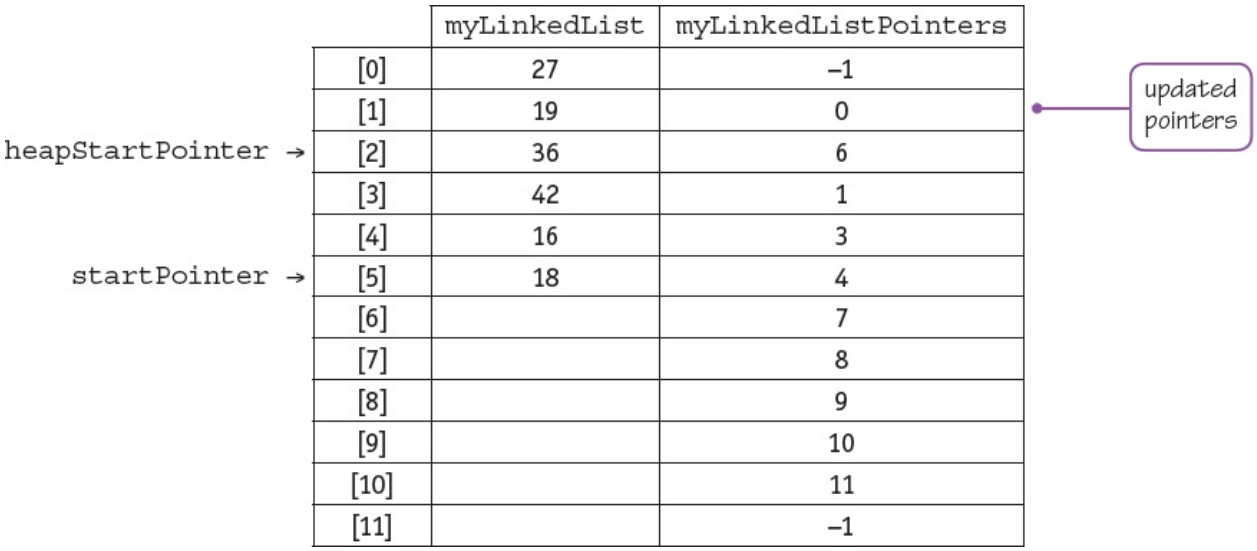


Figure 19.9

The following procedure deletes an item from a linked list.

Python

```
def delete(itemDelete):
    global startPointer, heapStartPointer
    if startPointer == nullPointer:
        print("Linked List empty")
    else:
        index = startPointer
        while myLinkedList[index] != itemDelete and index != nullPointer:
            oldindex = index
            index = myLinkedListPointers[index]
        if index == nullPointer:
            print("Item ", itemDelete, " not found")
        else:
            myLinkedList[index] = None
            tempPointer = myLinkedListPointers[index]
            myLinkedListPointers[index] = heapStartPointer
            heapStartPointer = index
            myLinkedListPointers[oldindex] = tempPointer
```

VB

```
Sub delete (ByVal itemDelete)
    Dim tempPointer, index, oldIndex  As Integer
    If startPointer = nullPointer Then
        Console.WriteLine("Linked List empty")
    Else
        index = startPointer
        While myLinkedList(index) <> itemDelete And index <> nullPointer
            Console.WriteLine( myLinkedList(index) & " " & index)
            Console.ReadKey()
            oldIndex = index
            index = myLinkedListPointers(index)
        End While
        if index = nullPointer Then
            Console.WriteLine("Item " & itemDelete & " not found")
        Else
            myLinkedList(index) = nothing
            tempPointer = myLinkedListPointers(index)
            myLinkedListPointers(index) = heapStartPointer
            heapStartPointer = index
            myLinkedListPointers(oldIndex) = tempPointer
        End If
    End If
End Sub
```

Java

```
static void delete(int itemDelete)
{
    int oldIndex = -1;
    if (startPointer == nullPointer)
        System.out.println("Linked List is empty");
    else
    {
        int index = startPointer;
        while (myLinkedList[index] != itemDelete && index != nullPointer)
        {
            oldIndex = index;
            index = myLinkedListPointers[index];
        }
        if (index == nullPointer)
            System.out.println("Item " + itemDelete + " not found");
        else
        {
            myLinkedList[index] = 0;
            int tempPointer = myLinkedListPointers[index];
            myLinkedListPointers[index] = heapStartPointer;
            heapStartPointer = index;
            myLinkedListPointers[oldIndex] = tempPointer;
        }
    }
}
```

ACTIVITY 19I

Use the algorithm to remove 16 from myLinkedList. Show this in a trace table and show myLinkedList once 16 has been removed. Add the delete procedure to your program, add code to input an item, delete this item to the linked list, then print out the list and the pointers before and after the item was deleted.

Binary trees

A **binary tree** is another frequently used ADT. It is a hierarchical data structure in which each parent node can have a maximum of two child nodes. There are many uses for binary trees; for example, they are used in syntax analysis, compression algorithms and 3D video games.

Figure 19.10 shows the binary tree for the data stored in myList sorted in ascending order. Each item is stored at a node and each node can have up to two branches with the rule if the value to be added is less than the current node branch left, if the value to be added is greater than or equal

to the current node branch right.

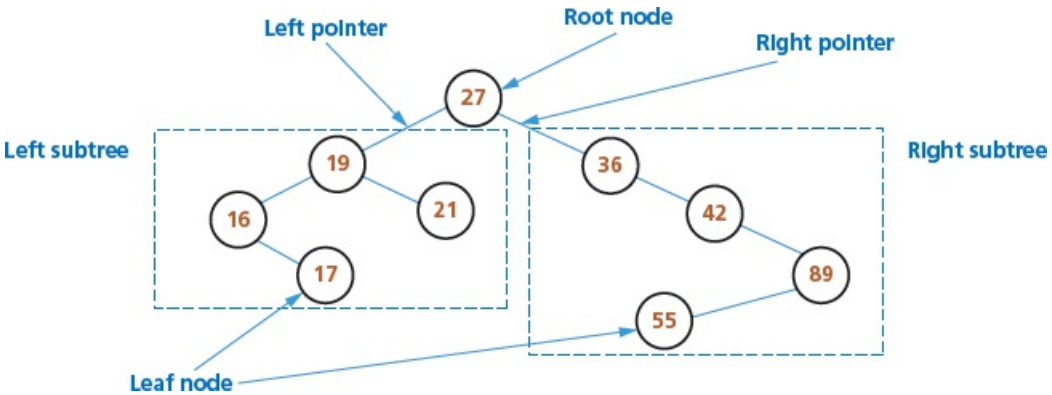


Figure 19.10 Example of an ordered binary tree

A binary tree can also be used to represent an arithmetic expression. Consider (a + b) * (c – a)

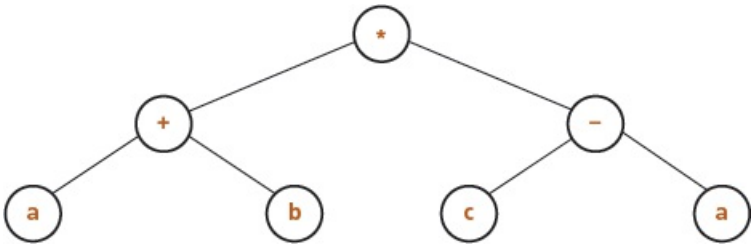


Figure 19.11 Example of an expression as a binary tree

ACTIVITY 19J

Draw the binary tree for the expression (x – y) / (x * y + z).

EXTENSION ACTIVITY 19B

Find out about different tree traversals and how they are used to convert an expression into reverse Polish.

The data structure for an ordered binary tree can be created in pseudocode as follows:

```
TYPE node
    DECLARE item : INTEGER
    DECLARE leftPointer : INTEGER
    DECLARE rightPointer : INTEGER
ENDTYPE
DECLARE myTree[0 : 8] OF node
DECLARE rootPointer : INTEGER
DECLARE nextFreePointer : INTEGER
```

ACTIVITY 19K

Create the data structure in pseudocode for a binary tree to store a list of names. Your list must be able to store at least 50 names.

The populated contents of the data structure myTree is shown below.

Root pointer

myTree	item	leftPointer	rightPointer
[0]	27	1	2
[1]	19	4	6
[2]	36	–1	3
[3]	42	–1	5
[4]	16	–1	7
[5]	89	8	–1
[6]	21	–1	–1
[7]	17	–1	–1
[8]	55	–1	–1

Pointers to items in the tree. –1 is used as a null pointer

Figure 19.12

The root pointer points to the first node in a binary tree. A null pointer is a value stored in the left or right pointer in a binary tree to indicate that there are no nodes below this node on the left or right.

Finding an item in a binary tree

The algorithm to find if an item is in the binary tree myTree and return the pointer to its node if found or a null pointer if not found, could be written as a function in pseudocode, as shown.

```
DECLARE rootPointer : INTEGER
DECLARE itemPointer : INTEGER
DECLARE itemSearch : INTEGER
CONSTANT nullPointer = -1
rootPointer ← 0
FUNCTION find(itemSearch) RETURNS INTEGER
itemPointer ← rootPointer
WHILE myTree[itemPointer].item <> itemSearch AND
  (itemPointer <> nullPointer) DO
    IF myTree[itemPointer].item > itemSearch
      THEN
        itemPointer ← myTree[itemPointer].leftPointer
      ELSE
        itemPointer ← myTree[itemPointer].rightPointer
    ENDIF
ENDWHILE
RETURN itemPointer
```

Here is the identifier table for the binary tree search algorithm shown above.

Identifier	**Description**
myTree	Tree to be searched
node	ADT for tree
rootPointer	Pointer to the start of the tree
leftPointer	Pointer to the left branch
rightPointer	Pointer to the right branch
nullPointer	Null pointer set to −1
itemPointer	Pointer to current item
itemSearch	Item being searched for

Table 19.20

The trace table below shows the algorithm being used to search for 42 in myTree.

rootPointer	itemPointer	itemSearch

0	0	42
	2	
	3	

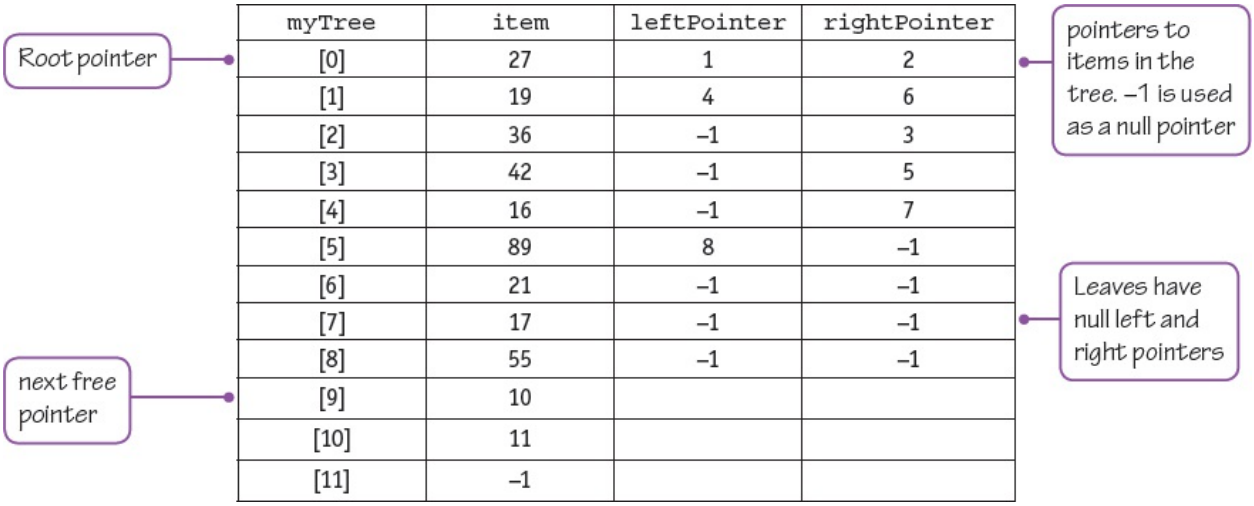
Table 19.21 Trace table

ACTIVITY 19L

Use the algorithm to search for 55 and 75 in myTree. Show the results of each search in a trace table.

Inserting items into a binary tree

The binary tree needs free nodes to add new items. For example, myTree, shown in Figure 19.13 below, now has room for 12 items. The last three nodes have not been filled yet, there is a pointer to the next free node and the free nodes are set up like a heap in a linked list, using the left pointer.



myTree	item	leftPointer	rightPointer
[0]	27	1	2
[1]	19	4	6
[2]	36	–1	3
[3]	42	–1	5
[4]	16	–1	7
[5]	89	8	–1
[6]	21	–1	–1
[7]	17	–1	–1
[8]	55	–1	–1
[9]	10		
[10]	11		
[11]	–1		

Figure 19.13

The algorithm to insert an item at a new node in the binary tree myTree could be written as a procedure in pseudocode as shown below.

```
TYPE node
    DECLARE item : INTEGER
    DECLARE leftPointer : INTEGER
    DECLARE rightPointer : INTEGER
    DECLARE oldPointer : INTEGER
    DECLARE leftBranch : BOOLEAN
ENDTYPE
DECLARE myTree[0 : 11] OF node
// binary tree now has extra spaces
DECLARE rootPointer : INTEGER
DECLARE nextFreePointer : INTEGER
DECLARE itemPointer : INTEGER
DECLARE itemAdd : INTEGER
DECLARE itemAddPointer : Integer
CONSTANT nullPointer = -1
// needed to use the binary tree
PROCEDURE nodeAdd(itemAdd)
    // check for full tree
    IF nextFreePointer = nullPointer
      THEN
        OUTPUT "No nodes free"
    ELSE
      //use next free node
      itemAddPointer ← nextFreePointer
      nextFreePointer ← myTree[nextFreePointer].leftPointer
      itemPointer ← rootPointer
      // check for empty tree
      IF itemPointer = nullPointer
        THEN
          rootPointer ← itemAddPointer
        ELSE
          // find where to insert a new leaf
          WHILE (itemPointer <> nullPointer) DO
              oldPointer ← itemPointer
              IF myTree[itemPointer].item > itemAdd
                THEN  // choose left branch
                  leftBranch ← TRUE
                  itemPointer ← myTree[itemPointer].leftPointer
                ELSE  // choose right branch
                  leftBranch ← FALSE
                  itemPointer ← myTree[itemPointer].rightPointer
              ENDIF
          ENDWHILE
          IF leftBranch  //use left or right branch
            THEN
              myTree[oldPointer].leftPointer ← itemAddPointer
            ELSE
              myTree[oldPointer].rightPointer ← itemAddPointer
          ENDIF
      ENDIF
      // store item to be added in the new node
      myTree[itemAddPointer].leftPointer ← nullPointer
      myTree[itemAddPointer].rightPointer ← nullPointer
      myTree[itemAddPointer].item ← itemAdd
    ENDIF
ENDPROCEDURE
```

Here is the identifier table.

Identifier	Description
myTree	Tree to be searched
node	ADT for tree
rootPointer	Pointer to the start of the tree
leftPointer	Pointer to the left branch
rightPointer	Pointer to the right branch
nullPointer	Null pointer set to -1
itemPointer	Pointer to current item in tree
itemAdd	Item to add to tree
nextFreePointer	Pointer to next free node
itemAddPointer	Pointer to position in tree to store item to be added
oldPointer	Pointer to leaf node that is going to point to item added
leftBranch	Flag to identify whether to go down the left branch or the right branch

Table 19.22

The trace table below shows the algorithm being used to add 18 to myTree.

leftBranch	nextFreePointer	itemAddPointer	rootPointer	itemAdd	itemPointer	oldPointer
	Already set to 9	9	Already set to 0	18		
	10				0	0
TRUE					1	1
TRUE					4	4
FALSE					7	7
					–1	

Table 19.23

The tree, myTree will now be as shown below.

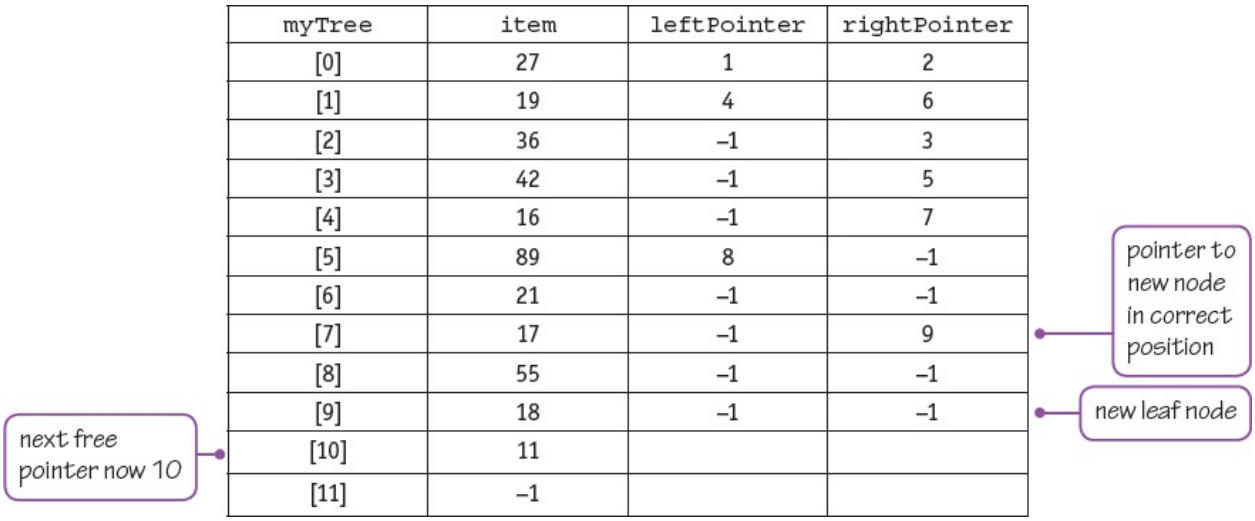


Figure 19.14

ACTIVITY 19M

Use the algorithm to add 25 to myTree. Show this in a trace table and show myTree once 25 has been added.

Implementing binary trees in Python, VB.NET or Java requires the use of objects and recursion. An example will be given in Chapter 20.

Graphs

A **graph** is a non-linear data structure consisting of nodes and edges. This is an ADT used to implement directed and undirected graphs. A graph consists of a set of nodes and edges that join a pair of nodes. If the edges have a direction from one node to the other it is a directed graph.

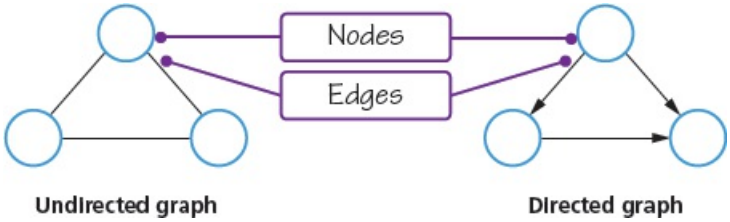


Figure 19.15

As we saw in Chapter 18, graphs are used to represent real life networks, such as

- bus routes, where the nodes are bus stops and the edges connect two stops next to each other
- websites, where each web page is a node and the edges show the links between each web page
- social media networks, where each node contains information about a person and the edges connect people who are friends.

Each edge may have a weight; for example, in the bus route, the weight could be the distance between bus stops or the cost of the bus fare.

A path is the list of nodes connected by edges between two given nodes and a cycle is a list of

nodes that return to the same node.

For example, a graph of the bus routes in a town could be as follows. The distance between each bus stop in kilometres is shown on the graph.

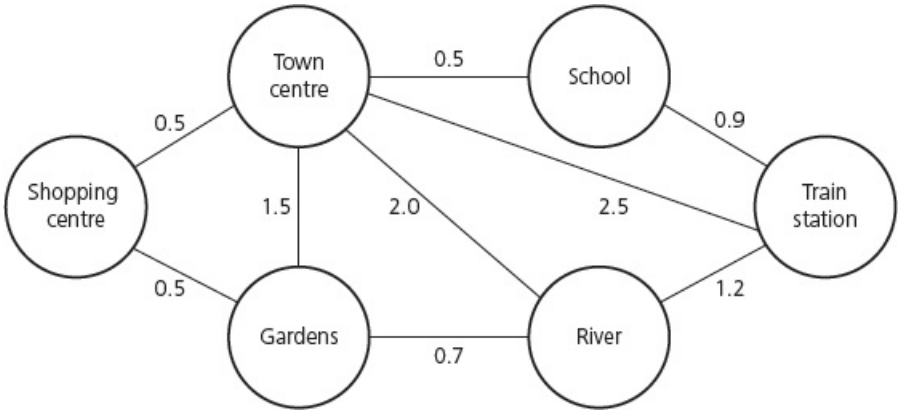


Figure 19.16

ACTIVITY 19N

Find another path from School to Gardens. Find the shortest path from Town centre to Train station. Find the shortest cycle from the Town centre.

A path from School to Gardens could be Path = (School, Train station, River, Gardens).

19.1.4 Implementing one ADT from another ADT

Every ADT is a collection of data and the methods used on the data. When an ADT is defined, the definition can refer to other data types. For example, myLinkedList refers to the data type INTEGER in its data definition.

A linked list type could be defined as follows.

```
TYPE linkedList
  DECLARE item : INTEGER
  DECLARE Pointer : INTEGER
ENDTYPE
// a linked list to store integers
```

And then used as follows.

```
DECLARE myLinkedList : ARRAY [0:11] OF linkedList
DECLARE heapStartPointer : INTEGER
DECLARE startPointer : INTEGER
DECLARE index : INTEGER
```

ACTIVITY 19O

Write pseudocode to declare a linked list to store names. Use this to write pseudocode to set up a linked list that will store 30 names. Write a program to store and display names in this linked list.

The data types for a stack, queue and a binary tree have been defined using existing data types.

Another data type is a **dictionary**, which is an ADT that consists of pairs consisting of a key and a value, where the key is used to find the value. Each key can only appear once. Keys in a dictionary are unordered. A value is retrieved from a dictionary by specifying its corresponding key. The same value may appear more than once. A dictionary differs from a set because the values can be duplicated. As a dictionary is not an ordered list, it can be declared using a linked list as part of the definition.

A dictionary type could be defined in pseudocode as follows.

```
TYPE linkedList
  DECLARE item : STRING
  DECLARE pointer : INTEGER
ENDTYPE
TYPE dictionary
  DECLARE key : myLinkedList : ARRAY [0:19] OF
  linkedList
  DECLARE value : ARRAY [0:19] OF STRING
ENDTYPE
```

And then used as follows.

```
DECLARE myDictionary : linkedList
DECLARE heapStartPointer : INTEGER
DECLARE startPointer : INTEGER
DECLARE index : INTEGER
```

Each of the programming languages used in Cambridge International A Level Computer Science provide a dictionary data type, as shown in the table below.

Dictionary data type example	Language

studentdict = {
"Leon": 27,
"Ahmad": 78,
"Susie": 64
}
	Python

Dim studentdict As New Dictionary(Of String, Integer) studentdict.Add("Leon", 27) studentdict.Add("Ahmad", 78) studentdict.Add("Susie", 64)	VB

studentdict = dict([("Leon", 27), ("Ahmad", 78), ("Susie", 64)])
Or
Dictionary<Integer, String> studentdict = new Hashtable<Integer, String>();
studentdict.put(27,"Leon");
studentdict.put(78,"Ahmad");
studentdict.put(64,"Susie");
	Java Dictionary is no longer used in Java but can be implemented using a hash table

Table 19.24

ACTIVITY 19P

In the programming language of your choice, write a program to use a dictionary to store the names of students as their keys and their examination scores as their values. Then find a student's examination score, add a student and score and delete a student and score.

19.1.5 Comparing algorithms

Big O notation is a mathematical notation used to describe the performance or complexity of an algorithm in relation to the time taken or the memory used for the task. It is used to describe the worst-case scenario; for example, how the maximum number of comparisons required to find a value in a list using a particular search algorithm increases with the number of values in the list.

Big O order of time complexity

	Description	Example
O(1)	describes an algorithm that always takes the same time to perform the task	deciding if a number is even or odd
O(N)	describes an algorithm where the time to perform the task will grow linearly in direct proportion to N, the number of items of data the algorithm is using	a linear search
$O(N^2)$	describes an algorithm where the time to perform the task will grow linearly in direct proportion to the square of N, the number of items of data the algorithm is using	bubble sort, insertion sort
$O(2^N)$	describes an algorithm where the time to perform the task doubles every time the algorithm uses an extra item of data	calculation of Fibonacci numbers using recursion (see Section 19.2)
O(Log N)	describes an algorithm where the time to perform the task goes up linearly as the number of items goes up exponentially	binary search

Table 19.25 Big O order of time complexity

Big O order of space complexity

	Description	Example
O(1)	describes an algorithm that always uses the same space to perform the task	any algorithm that just uses variables, for example d = a + b + c
O(N)	describes an algorithm where the space to perform the task will grow linearly in direct proportion to N, the number of items of data the algorithm is using	any algorithm that uses arrays, for example a loop to calculate a running total of values input to an array of N elements

Table 19.26 Big O order of space complexity

ACTIVITY 19Q

1 Using diagrams, describe the structure of
   a) a binary tree
   b) a linked list.
2 a) Explain what is meant by a *dictionary data type*.
   b) Show how a dictionary data type can be constructed from a linked list.
3 Compare the performance of a linear search and a binary search using Big O notation.

19.2 Recursion

WHAT YOU SHOULD ALREADY KNOW

Remind yourself of the definitions of the following mathematical functions, which many of you will be familiar with, and see how they are constructed.

- Factorials
- Arithmetic sequences
- Fibonacci numbers
- Compound interest

Key terms

Recursion – a process using a function or procedure that is defined in terms of itself and calls itself.

Base case – a terminating solution to a process that is not recursive.

General case – a solution to a process that is recursively defined.

Winding – process which occurs when a recursive function or procedure is called until the base case is found.

Unwinding – process which occurs when a recursive function finds the base case and the function returns the values.

19.2.1 Understanding recursion

Recursion is a process using a function or procedure that is defined in terms of itself and calls itself. The process is defined using a **base case**, a terminating solution to a process that is not recursive, and a **general case**, a solution to a process that is recursively defined.

For example, a function to calculate a factorial for any positive whole number n! is recursive. The definition for the function uses:

a base case of 0! = 1

a general case of n! = n * (n–1)!

This can be written in pseudocode as a recursive function.

```
FUNCTION factorial (number : INTEGER) RETURNS INTEGER
    IF number = 0
      THEN
        answer ← 1 // base case
      ELSE
        answer ← number * factorial (number - 1)
        // recursive call with general case
    ENDIF
    RETURN answer
ENDFUNCTION
```

With recursive functions, the statements after the recursive function call are not executed until the base case is reached; this is called **winding**. After the base case is reached and can be used in the recursive process, the function is **unwinding**.

In order to understand how the winding and unwinding processes in recursion work, we can use a trace table for a specific example: 3!

Call number	Function call	number	answer	RETURN
1	Factorial (3)	3	3 * factorial (2)	
2	Factorial (2)	2	2 * factorial (1)	
3	Factorial (1)	1	1 * factorial (0)	
4	Factorial (0)	0	1	1
3 continued	Factorial (1)	1	1 * 1	1
2 continued	Factorial (2)	2	2 * 1	2
1 continued	Factorial (3)	3	3 * 2	6

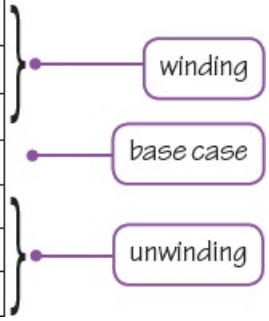


Table 19.27

Here is a simple recursive factorial program written in Python, VB and Java using a function.

Python

```
#Python program recursive factorial function
def factorial(number):
    if number == 0:
        answer = 1
    else:
        answer = number * factorial(number - 1)
    return answer
print(factorial(0))
print(factorial(5))
```

VB

```
'VB program recursive factorial function
Module Module1
    Sub Main()
        Console.WriteLine(factorial(0))
        Console.Writeline(factorial(5))
        Console.ReadKey()
    End Sub
    Function factorial(ByVal number As Integer) As Integer
           Dim answer As Integer
        If number = 0 Then
           answer = 1
       Else
            answer = number * factorial(number - 1)
         End If
                return answer
    End Function
End Module
```

Java

```
// Java program recursive factorial function
public class Factorial {
    public static void main(String[] args) {
        System.out.println(factorial(0));
        System.out.println(factorial(5));
    }
    public static int factorial(int number)
    {
        int answer;
            if (number == 0)
                    answer = 1;
        else
                    answer = number * factorial(number - 1);
        return answer;
    }
}
```

ACTIVITY 19R

Write the recursive factorial function in the programming language of your choice. Test your program with 0! and 5!

Complete trace tables for 0! and 5! using the recursive factorial function written in pseudocode and compare the results from your program with the trace tables.

Compound interest can be calculated using a recursive function. Where the principal is the amount of money invested, rate is the rate of interest and years is the number of years the money has been invested.

The base case is	total0 = principal where years = 0
The general case is	totaln = totaln-1 * rate

Table 19.28

```
DEFINE FUNCTION compoundInt(principal, rate, years : REAL) RETURNS REAL
    IF years = 0
      THEN
        total ← principal
      ELSE
        total ← compoundInt(principal * rate, rate, years - 1)
    ENDIF
    RETURN total
ENDFUNCTION
```

This function can be traced for a principal of 100 over three years at 1.05 (5% interest).

Call number	Function call	years	total	RETURN
1	compoundInt(100, 1.05, 3)	3	compoundInt(105, 1.05, 2)	
2	compoundInt(105, 1.05, 2)	2	compoundInt(105, 1.05, 1)	
3	compoundInt(105, 1.05, 1)	1	compoundInt(105, 1.05, 0)	
4	compoundInt(105, 1.05, 0)	0	100	100
3 cont	compoundInt(105, 1.05, 1)	1	105	105
2 cont	compoundInt(105, 1.05, 2)	2	110.25	110.25
1 cont	compoundInt(105, 1.05, 3)	3	115.76	115.76

Table 19.29

ACTIVITY 19S

The Fibonacci series is defined as a sequence of numbers in which the first two numbers are 0 and 1, depending on the selected beginning point of the sequence, and each subsequent number is the sum of the previous two.

Identify the base case and the general case for this series. Write a pseudocode algorithm to find and output the nth term. Test your algorithm by drawing a trace table for the fourth term.

EXTENSION ACTIVITY 19C

Write your function from Activity 19S in the high-level programming language of your choice. Test this with the 5th and 27th terms.

Benefits of recursion

Recursive solutions can contain fewer programming statements than an iterative solution. The solutions can solve complex problems in a simpler way than an iterative solution. However, if recursive calls to procedures and functions are very repetitive, there is a very heavy use of the stack, which can lead to stack overflow. For example, factorial(100) would require 100 function calls to be placed on the stack before the function unwinds.

19.2.2 How a compiler implements recursion

Recursive code needs to make use of the stack; therefore, in order to implement recursive procedures and functions in a high-level programming language, a compiler must produce object code that pushes return addresses and values of local variables onto the stack with each recursive call, winding. The object code then pops the return addresses and values of local variables off the stack, unwinding.

ACTIVITY 19T

1 Explain what is meant by *recursion* and give the benefits of using recursion in programming.

2 Explain why a compiler needs to produce object code that uses the stack for a recursive procedure.

End of chapter questions

1 Data is stored in the array NameList[1:10]. This data is to be sorted.

a) i) Copy and complete this pseudocode algorithm for an insertion sort.

[7]

```
FOR ThisPointer ← 2 TO ..........................................
    // use a temporary variable to store item which is to
    // be inserted into its correct location
    Temp ← NameList[ThisPointer]
    Pointer ← ThisPointer - 1
    WHILE (NameList[Pointer] > Temp) AND ....................
        // move list item to next location
        NameList[.....................] ← NameList[.....................]
        Pointer ← ..............................................
    ENDWHILE
    // insert value of Temp in correct location
    NameList[.................................] ← ........................
ENDFOR
```

ii) A special case is when NameList is already in order. The algorithm in part a) i) is applied to this special case.
Explain how many iterations are carried out for each of the loops.

[3]

b) An alternative sort algorithm is a bubble sort:

```
FOR ThisPointer ← 1 TO 9
    FOR Pointer ← 1 TO 9
        IF NameList[Pointer] > NameList[Pointer + 1]
          THEN
            Temp ← NameList[Pointer]
            NameList[Pointer] ← NameList[Pointer + 1]
            NameList[Pointer + 1] ← Temp
        ENDIF
    ENDFOR
ENDFOR
```

i) As in part a) ii), a special case is when NameList is already in order. The algorithm in part b) is applied to this special case.
Explain how many iterations are carried out for each of the loops.

[2]

ii) Rewrite the algorithm in part b), using **pseudocode**, to reduce the number of unnecessary comparisons.
Use the same variable names where appropriate.

[5]

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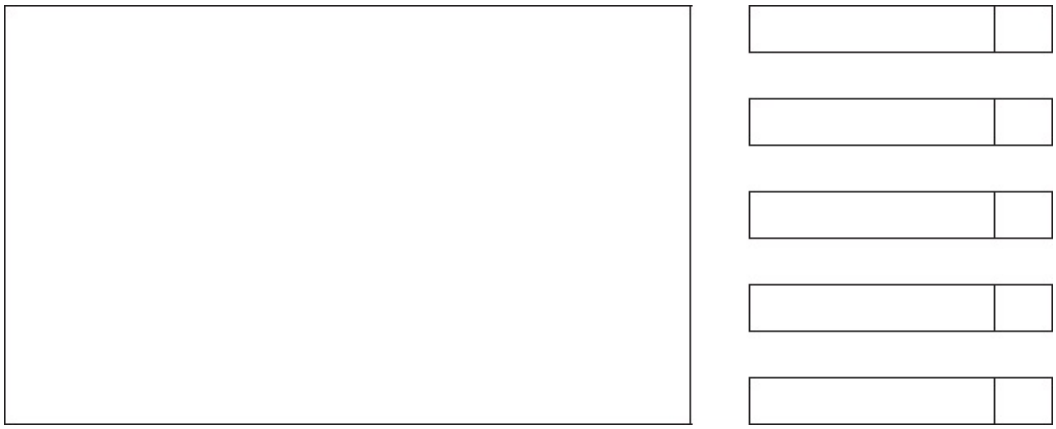
2 A Queue Abstract Data type (ADT) has these associated operations:
– create queue
– add item to queue
– remove item from queue
The queue ADT is to be implemented as a linked list of nodes.
Each node consists of data and a pointer to the next node.

a) The following operations are carried out:

```
CreateQueue
AddName("Ali")
AddName("Jack")
AddName("Ben")
AddName("Ahmed")
RemoveName
AddName("Jatinder")
RemoveName
```

Copy the diagram and add appropriate labels to show the final state of the queue. Use the space on the left as a workspace.
Show your final answer in the node shapes on the right.

[3]



b) Using pseudocode, a record type, Node, is declared as follows:

```
TYPE Node
    DECLARE Name    : STRING
    DECLARE Pointer : INTEGER
ENDTYPE
```

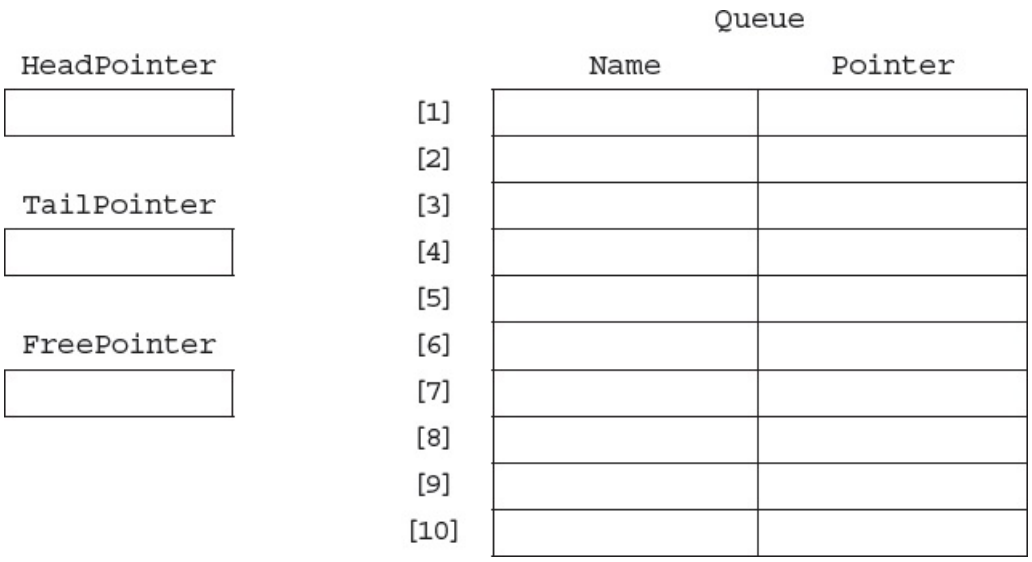
The statement

```
DECLARE Queue : ARRAY[1:10] OF Node
```

reserves space for 10 nodes in array Queue.

i) The CreateQueue operation links all nodes and initialises the three pointers that need to be used: HeadPointer, TailPointer and FreePointer.
Copy and complete the diagram to show the value of all pointers after CreateQueue has been executed.

[4]



ii) The algorithm for adding a name to the queue is written, using pseudocode, as a procedure with the header:

```
PROCEDURE AddName(NewName)
```

where NewName is the new name to be added to the queue.
The procedure uses the variables as shown in the identifier table.

Identifier	**Data type**	**Description**
Queue	Array[1:10] OF Node	Array to store node data
NewName	STRING	Name to be added
FreePointer	INTEGER	Pointer to next free node in array
HeadPointer	INTEGER	Pointer to first node in queue
TailPointer	INTEGER	Pointer to last node in queue
CurrentPointer	INTEGER	Pointer to current node

```
PROCEDURE AddName(BYVALUE NewName : STRING)
   // Report error if no free nodes remaining
   IF FreePointer = 0
     THEN
       Report Error
     ELSE
       // new name placed in node at head of
       free list
        CurrentPointer ← FreePointer
        Queue[CurrentPointer].Name ← NewName
       // adjust free pointer
       FreePointer ← Queue[CurrentPointer].
       Pointer
        // if first name in queue then adjust
        head pointer
        IF HeadPointer = 0
         THEN
           HeadPointer ← CurrentPointer
      ENDIF
     // current node is new end of queue
     Queue[CurrentPointer].Pointer ← 0
     TailPointer ← CurrentPointer
   ENDIF
ENDPROCEDURE
```

Copy and complete the **pseudocode** for the procedure RemoveName. Use the variables listed in the identifier table.

[6]

```
    PROCEDURE RemoveName()
     // Report error if Queue is empty
.....
.....
.....
.....
     OUTPUT Queue[.....].Name
     // current node  is head of queue
.....
     // update head pointer
.....
     // if only one element in queue then update tail
        pointer
.....
.....
.....
.....
     // link released node to free list
.....
.....
.....
    ENDPROCEDURE
```

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